Data structures for 3D Meshes

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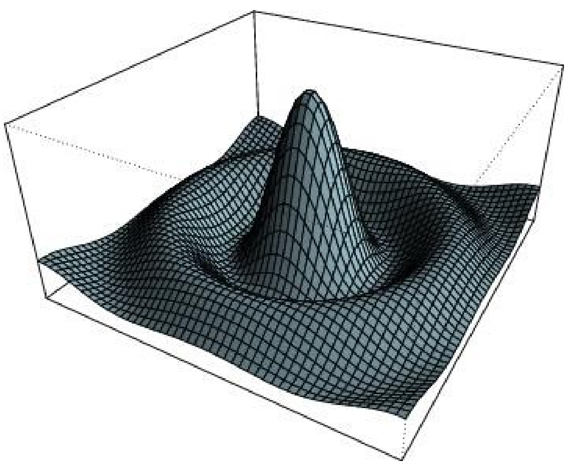
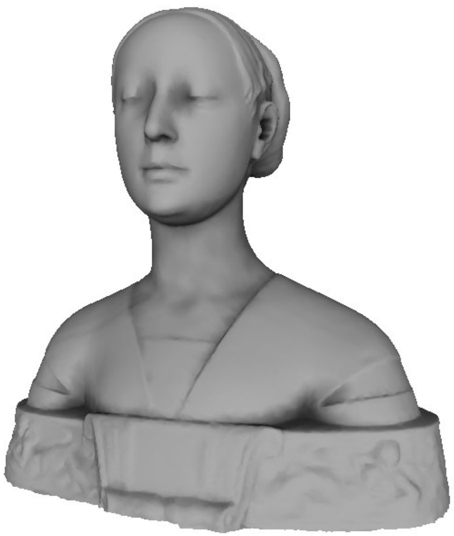
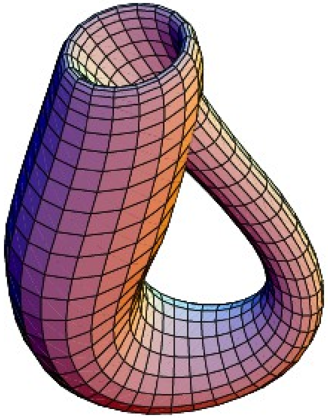
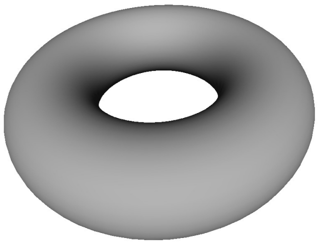
Surfaces

- A 2-dimensional region of 3D space
- *A portion of space having length and breadth but no thickness*







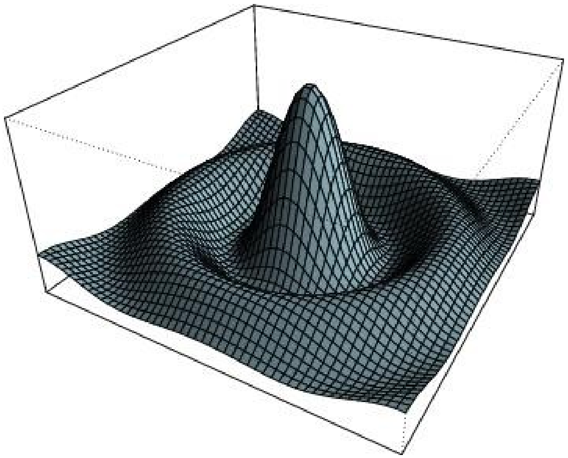


Defining Surfaces

- Analytically...
  - Parametric surfaces

$$S: R^2 \rightarrow R^3$$

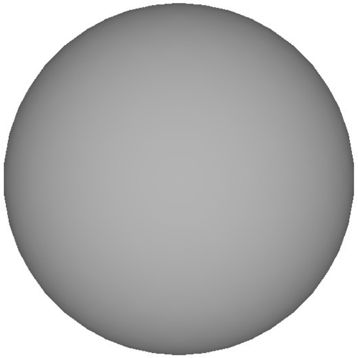
$$S(x,y) = \left(x, y, \sin\left(\sqrt{(x^2+y^2)}\right) / \sqrt{(x^2+y^2)}\right)$$



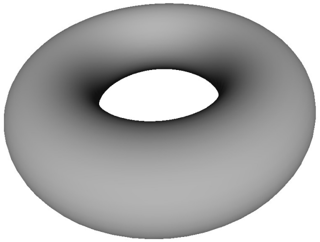
  - Implicit surfaces

$$S = \{(x,y,z): f(x,y,z) = 0\}$$

$$S = \{(x,y,z): x^2 + y^2 + z^2 - r^2 = 0\}$$



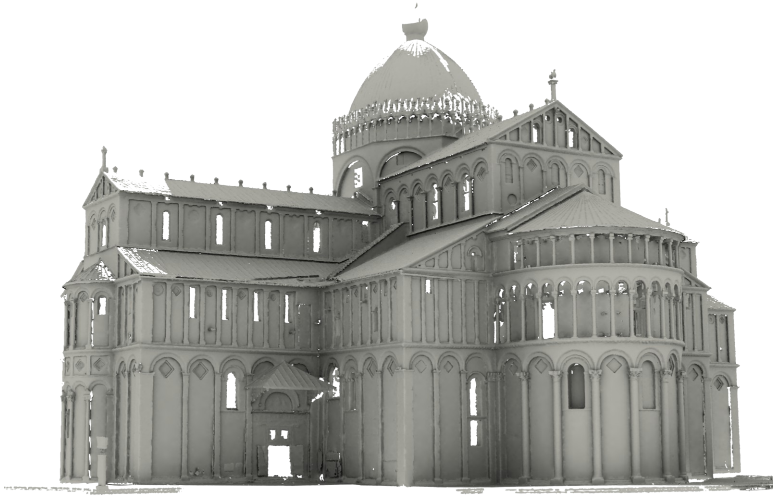
$$S = \{(x,y,z): (x^2 + y^2 + R^2 - r^2)^2 - 4R^2(x^2+y^2) = 0\}$$



Representing Real World Surfaces

- Analytic definition falls short of representing *real world* surfaces in a *tractable* way

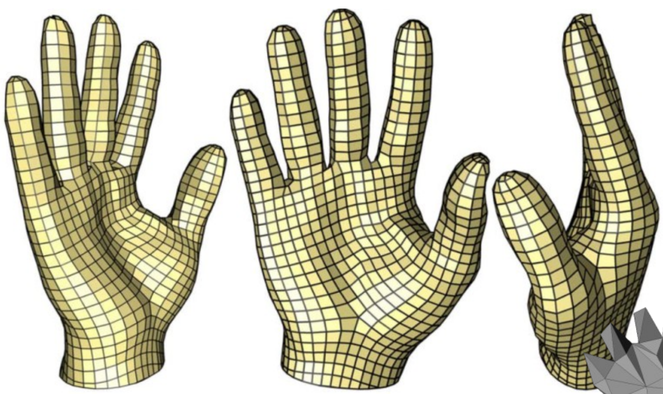
$$S(x, y) = \ldots.?$$



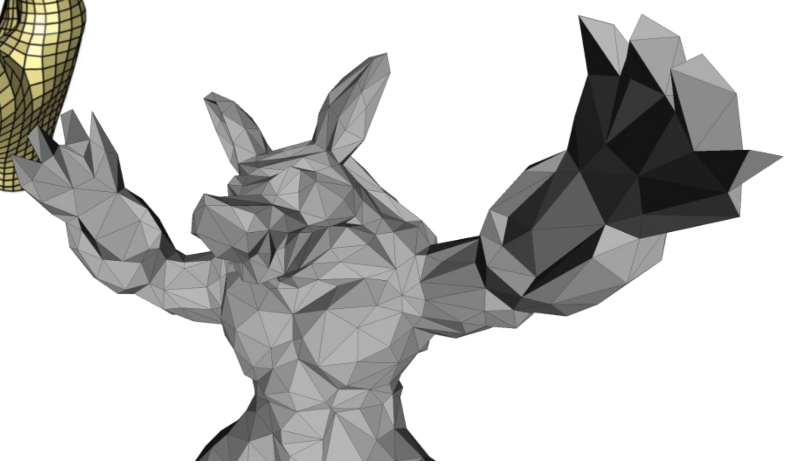
... surfaces can be represented by ***cell complexes***

Cell complexes (meshes)

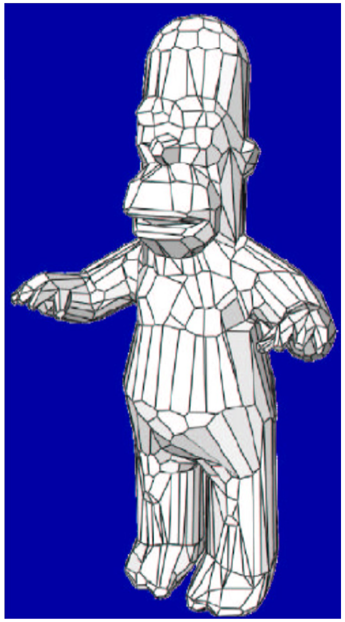
- Intuitive description: a continuous surface divided in polygons



quadrilaterals (quads)



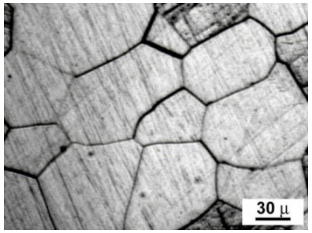
triangles

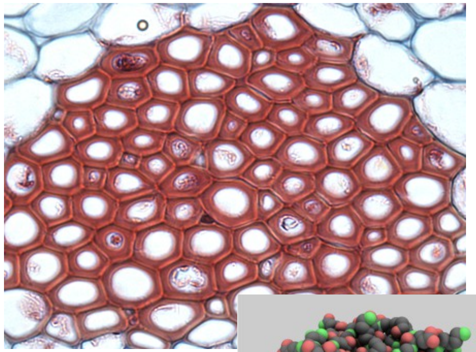


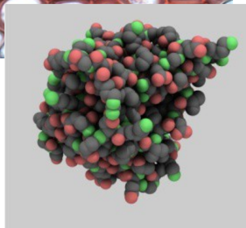
Generic polygons

Cell Complexes (meshes)

- In nature, meshes arise in a variety of contexts:
  - Cells in organic tissues
  - Crystals
  - Molecules
  - …
  - Mostly *convex* but *irregular* cells
  - Common concept: *complex* shapes can be described as *collections* of *simple building blocks*







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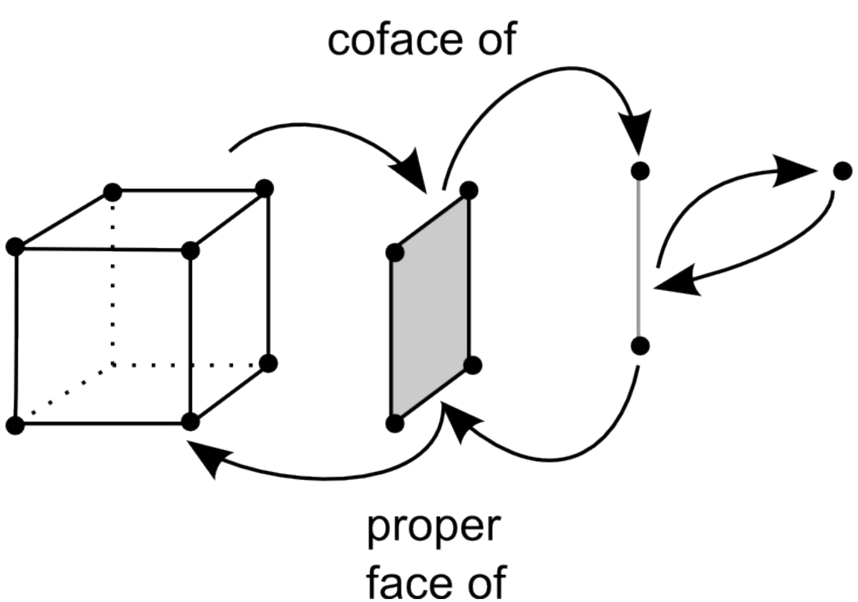
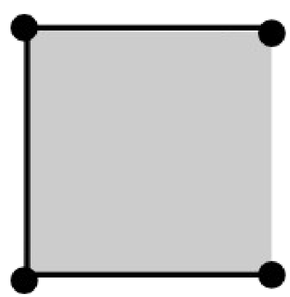
Cell Complexes (meshes)

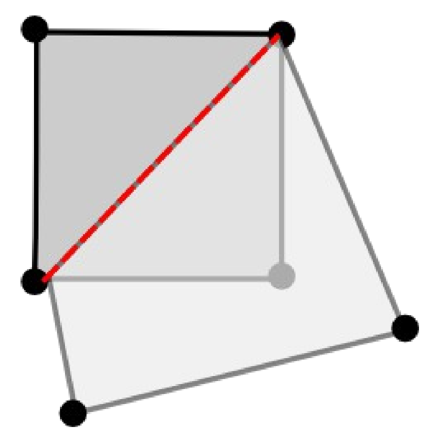
- more formal definition
  - a *cell* is a convex polytope in
  - a *proper face* of a cell is a convex polytope in



Cell Complexes (meshes)

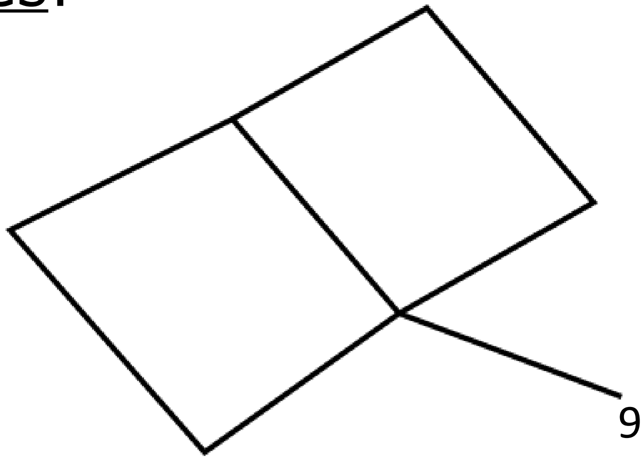
- a collection of cells is a complex **iff**
  - every face of a cell belongs to the comple
  - For every cells C and C′, their intersection either is empty or is a common face of bo





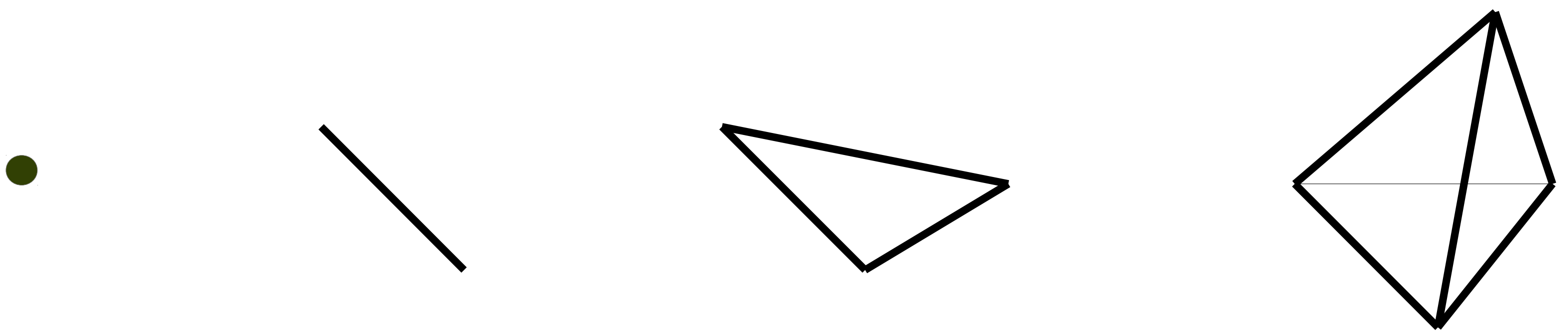
Maximal Cell Complex

- the **order** of a cell is the number of its sides (or vertices)
- a complex is a **k-complex** if the maximum of the order of its cells is *k*
- a cell is **maximal** if it is not a face of another cell
- a k-complex is **maximal** *iff* all maximal cells have order k
- short form : no dangling edges!



Simplicial Complex

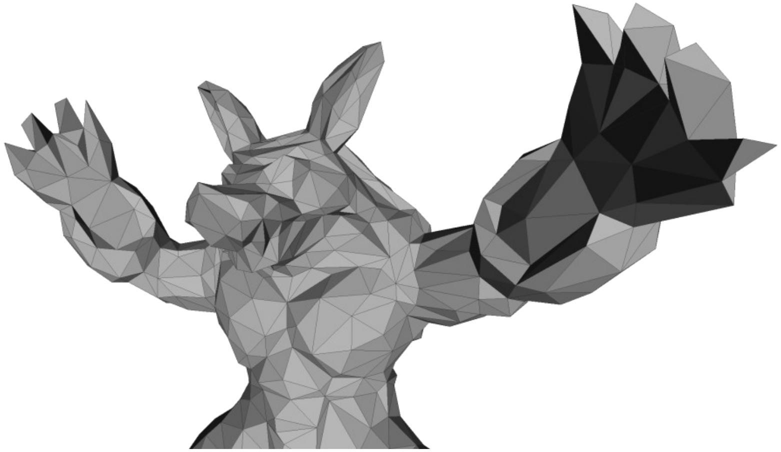
- A cell complex is a **simplicial complex** when the cells are simplexes
- A ***d-simplex*** is the convex hull of *d+1* points in



0-simplex 1-simplex 2-simplex 3-simplex

Meshes, at last

- When talking of *triangle mesh* the intended meaning is a **maximal 2-simplicial complex**



Topology vs Geometry

- Di un complesso simpliciale e' buona norma distinguere
  - Realizzazione geometrica
    - Dove stanno effettivamente nello spazio i vertici del nostro complesso simpliciale
  - Caratterizzazione topologica
    - Come sono connessi combinatoriamente i vari elementi

Topology vs geometry 2

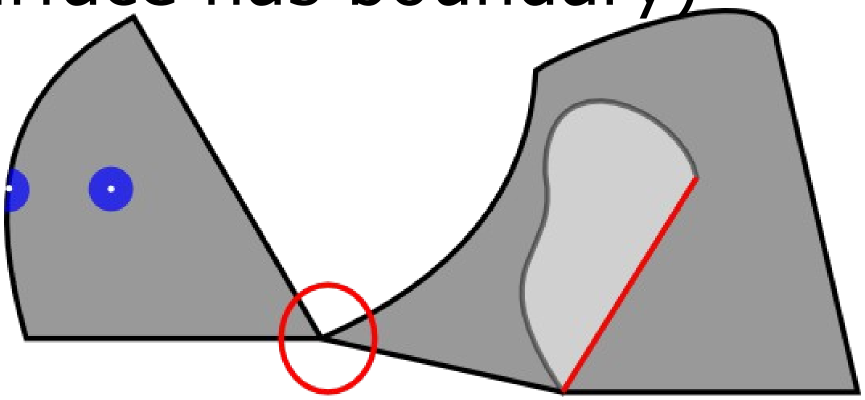
Nota: Di uno stesso oggetto e' possibile dare rappresentazioni con eguale realizzazione geometrica ma differente caratterizzazione topologica (molto differente!) Demo kleine

Nota: Di un oggetto si puo' dire molte cose considerandone solo la componente topologica

- Orientabilita
- componenti connesse
- bordi

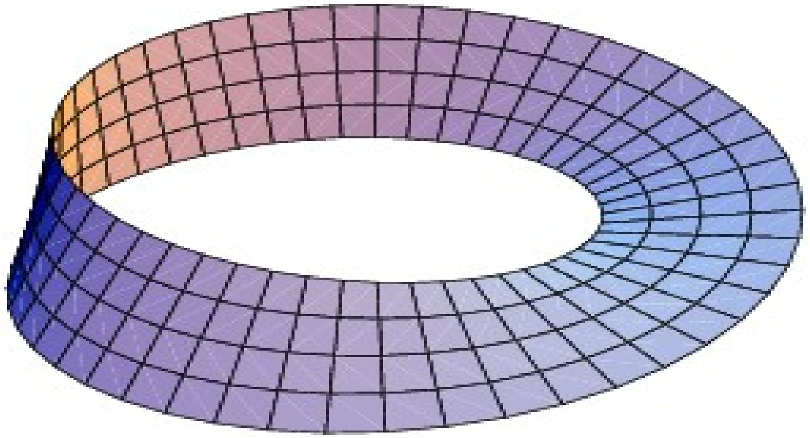
Manifoldness

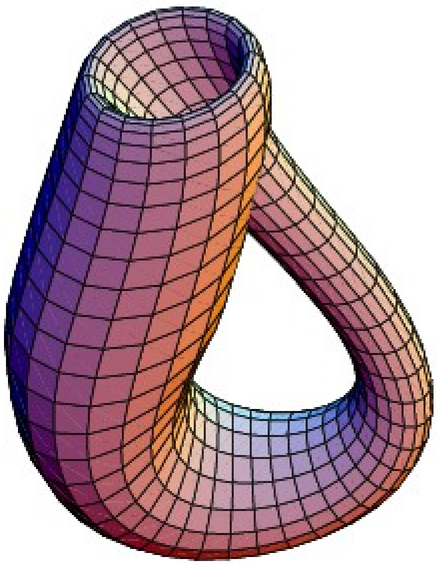
- a surface S is **2-manifold** *iff:*
  - the neighborhood of each point is homeomorphic to Euclidean space in two dimension
    or … in other words..
  - the neighborhood of each point is homeomorphic to a disk (or a semidisk if the surface has boundary)



Orientability

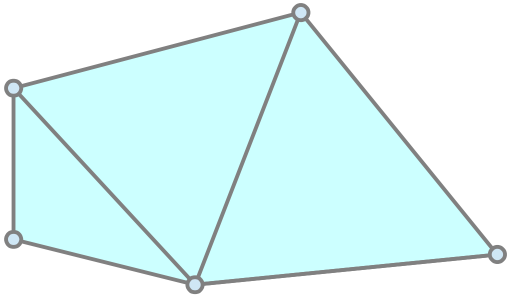
- A surface is **orientable** if it is possible to make a consistent choice for the normal vector
  - …it has two sides
- Moebius strips, klein bottles, and non manifold surfaces are not orientable





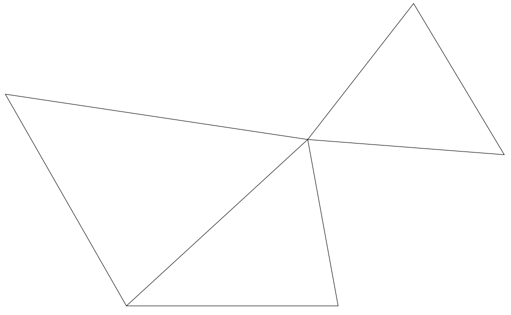
Incidenza Adiacenza

- Due simplessi σ e σ' sono incidenti se σè una faccia propria di σ' o vale il viceversa.
- Due k-simplessi sono m-adiacenti (k>m) se esiste un m-simplesso che è una faccia propria di entrambi.
  - Due triangoli che condividono un edge sono 1 -adiacenti
  - Due triangoli che condividono un vertice sono 0-adiacenti



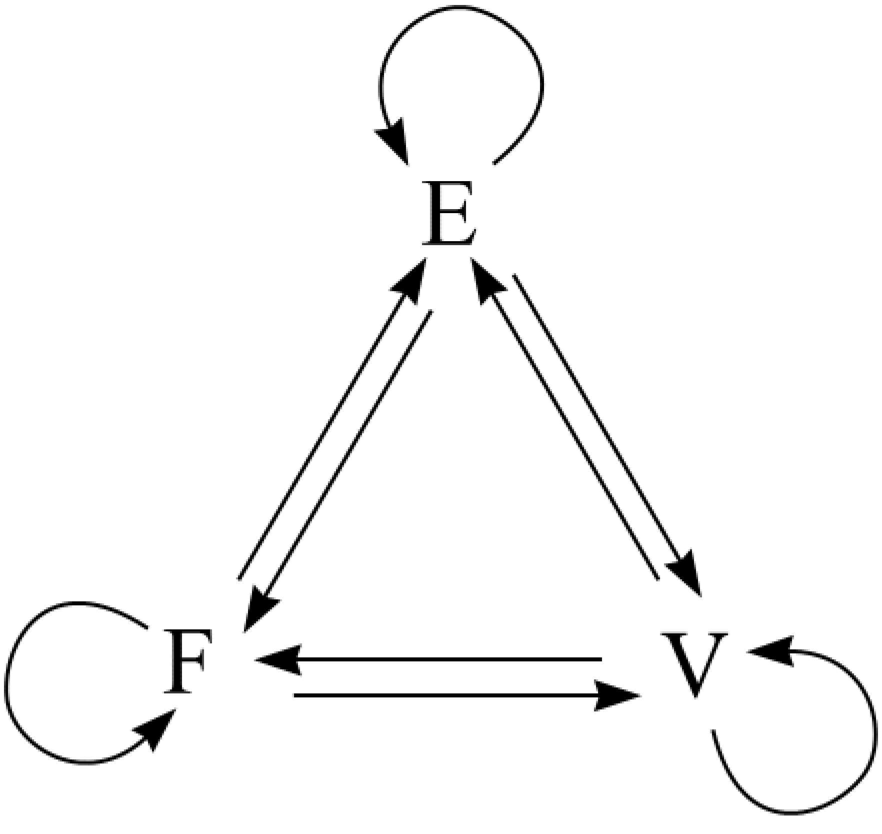
Relazioni di Adiacenza

- Per semplicità nel caso di mesh si una relazione di adiacenza con un una coppia (ordinata!) di lettere che indicano le entità coinvolte
  - FF adiacenza tra triangoli
  - FV i vertici che compongono un triangolo
  - VF i triangoli incidenti su un dato vertice



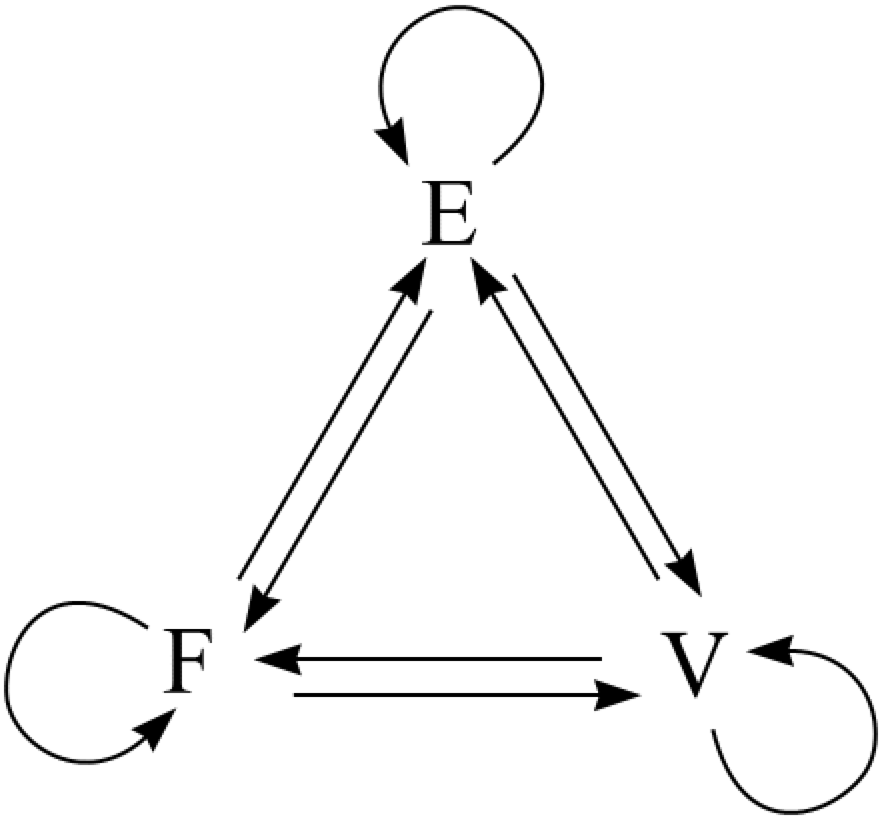
Relazioni di adiacenza

- Di tutte le possibili relazioni di adiacenza di solito vale la pena se ne considera solo un sottoinsieme (su 9) e ricavare le altre proceduralmente



Relazioni di adiacenza

- FF ~ 1-adiacenza
- EE ~ 0 adiacenza
- FE ~ sottofacce proprie di F con dim 1
- FV ~ sottofacce proprie di F con dim 0
- EV ~ sottofacce proprie di E con dim 0
- VF ~ F in Σ : V sub faccia di F
- VE ~ E in Σ : V sub faccia di E
- EF ~ F in Σ : E sub faccia di F
- VV ~ V' in Σ : Esiste E (V,V')



Partial adiacency

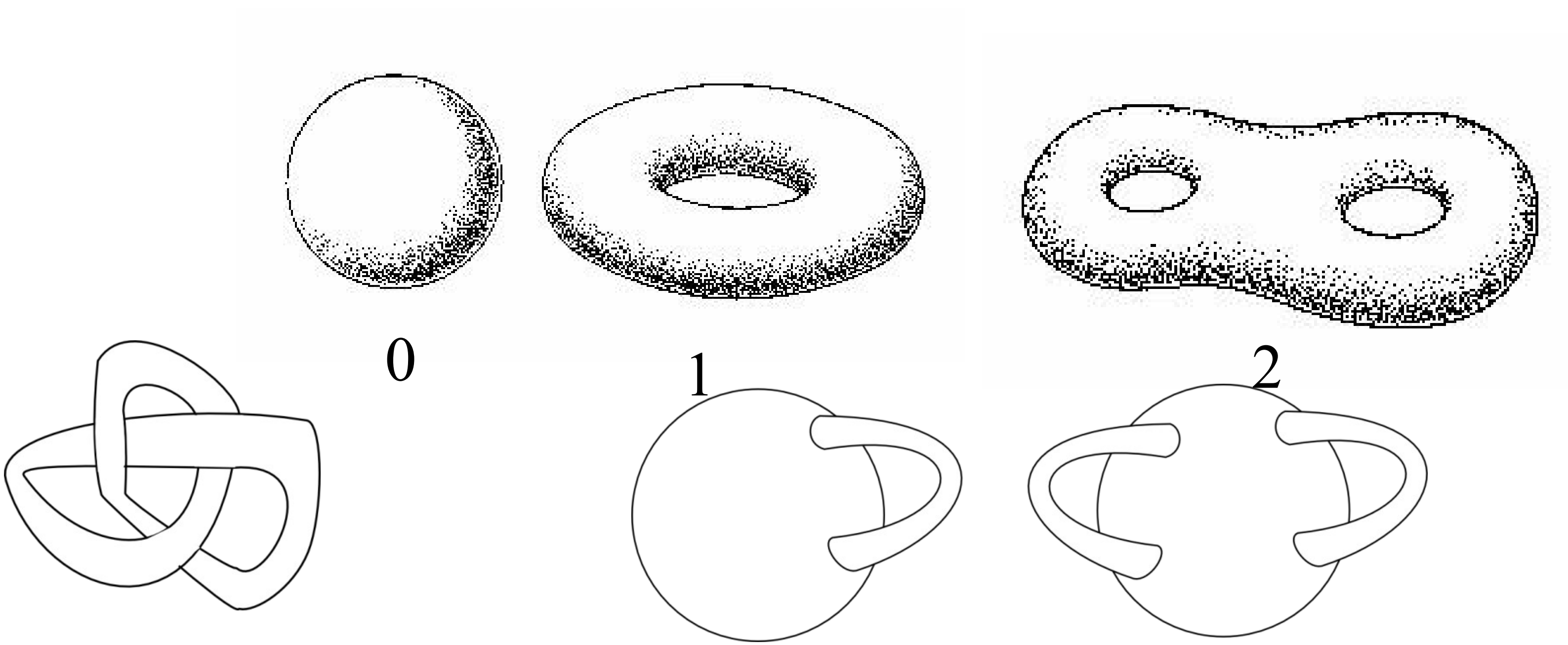
- Per risparmiare a volte si mantiene una informazione di adiacenza parziale
  - VF* memorizzo solo un riferimento dal vertice ad una delle facce e poi 'navigo' sulla mesh usando la FF per trovare le altre facce incidenti su V

Relazioni di adiacenza

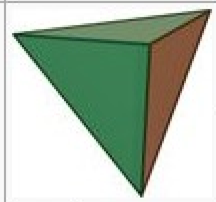
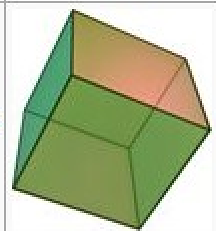
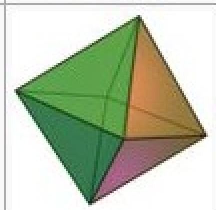
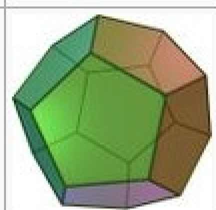
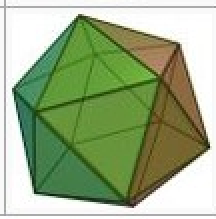
- In un 2-complesso simpliciale immerso in R3, che sia 2 manifold
  - FV FE FF EF EV sono di cardinalità bounded (costante nel caso non abbia bordi)
    - $|FV| = 3$ $|EV| = 2$ $|FE| = 3$
    - $|FF| <= 2$
    - $|EF| <= 2$
  - VV VE VF EE sono di card. variabile ma in stimabile in media
    - $|VV| \sim |VE| \sim |VF| \sim 6$
    - $|EE| \sim 10$
    - $F \sim 2V$

Genus

- The **Genus** of a closed surface, orientable and 2-manifold is the maximum number of cuts we can make along non intersecting closed curves without splitting the surface in two.

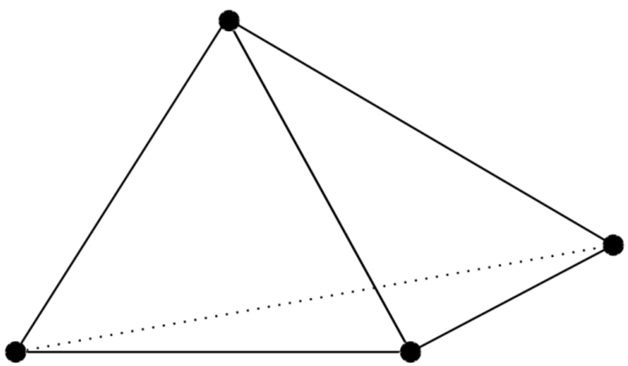


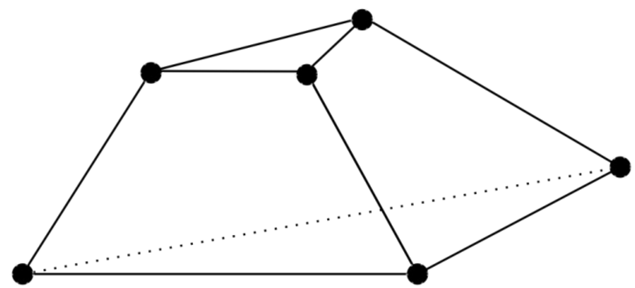
- …also known as the number of *handles*

Name	Image	*V* (vertices)	*E* (edges)	*F* (faces)	Euler characteristic: *V* − *E* + *F*
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

Euler characteristics

- χ= 2 for any *simply connected* polyhedron
- proof by construction…
- play with examples:



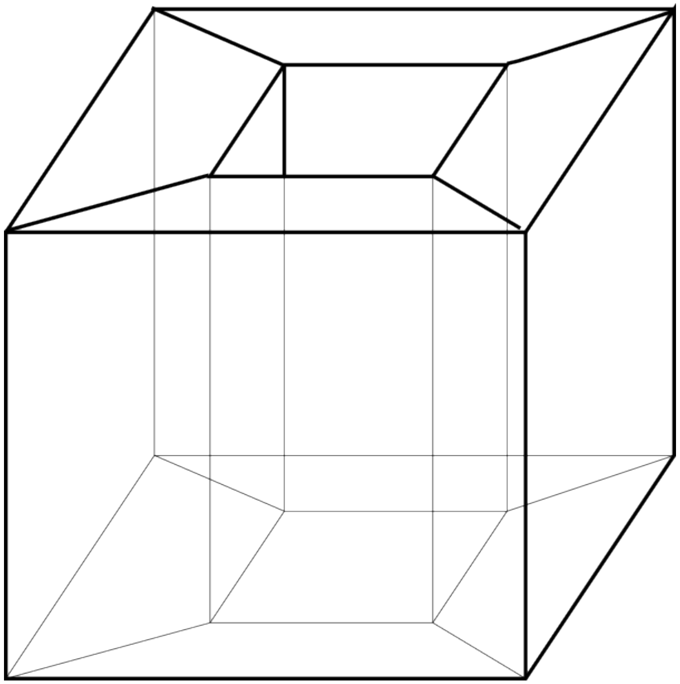


$$\chi = V - E + F$$
$$\chi = 4 - 6 + 4 = 2$$

$$\chi = (V + 2) - (E + 3) + (F + 1) =$$
$$\chi = (4 + 2) - (6 + 3) + (4 + 1) = 2$$

Euler characteristics

- let's try a more complex figure…



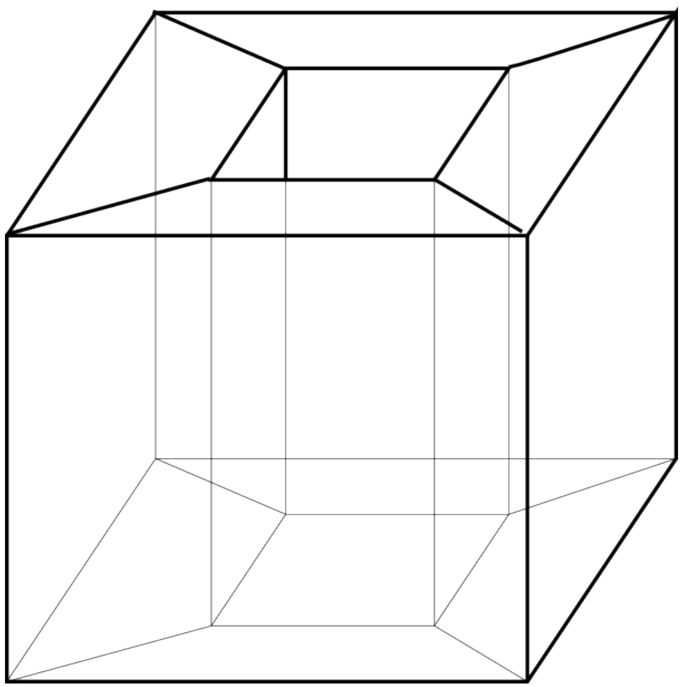
$$\chi = V - E + F$$
$$\chi = 16 - 32 + 16 = \mathbf{0}$$

- why =0 ?

Euler characteristics

$$\chi = 2 - 2g$$

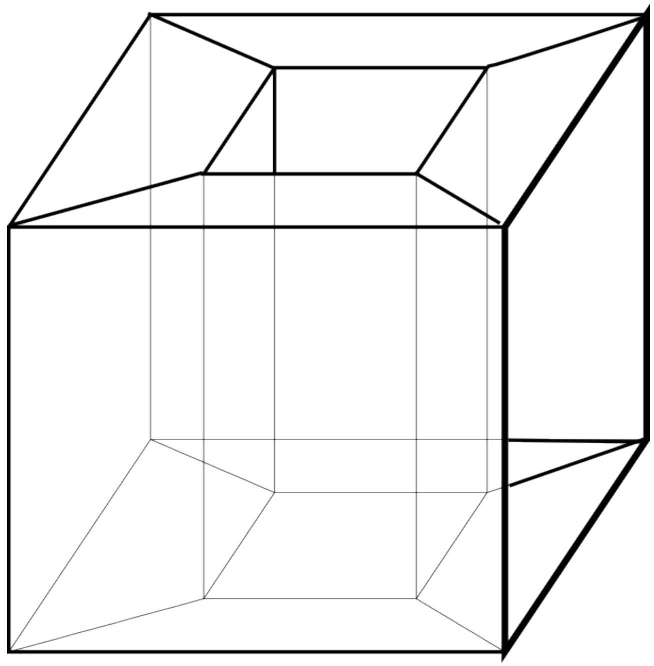
- where g is the genus of the surface



$$\chi = V - E + F$$
$$\chi = 16 - 32 + 16 = 0 = 2 - 2g$$

Euler characteristics

- let's try a more complex figure…remove a face. The surface is not closed anymore



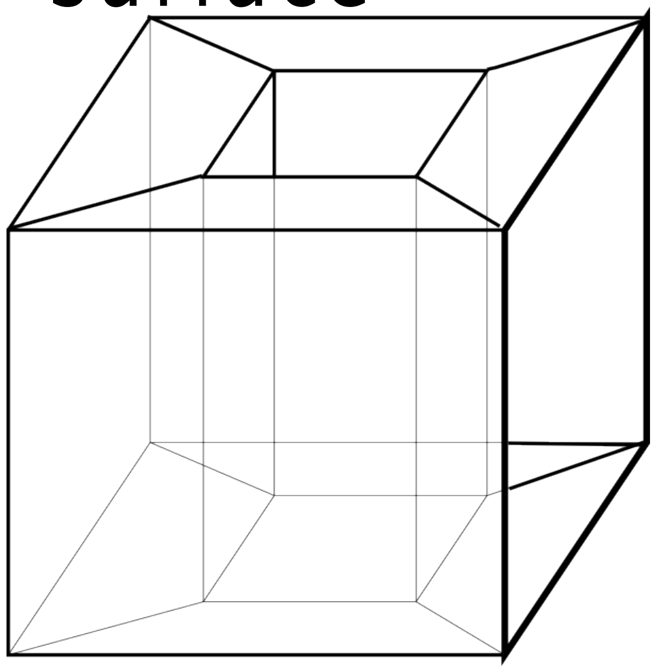
$$\chi = V - E + F$$
$$\chi = 16 - 32 + 15 = -\mathbf{1}$$

- why =-1 ?

Euler characteristics

$$\chi = 2 - 2g - b$$

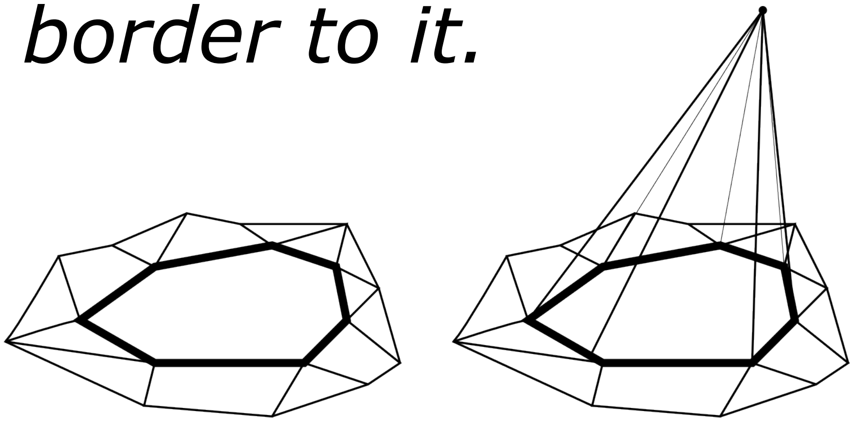
- where b is the number of borders of the surface



$$\chi = V - E + F$$
$$\chi = 16 - 32 + 15 = -1 = 2 - 2g - b$$

Euler characteristics

- *Remove the border by adding a new vertex and connecting all the **k** vertices on the border to it.*



A A'

$$X' = X + V' - E' + F' = X + 1 - k + k = X + 1$$

Differential quantities:normals

- The (unit) **normal** to a point is the (unit) vector perpendicular to the tangent plane

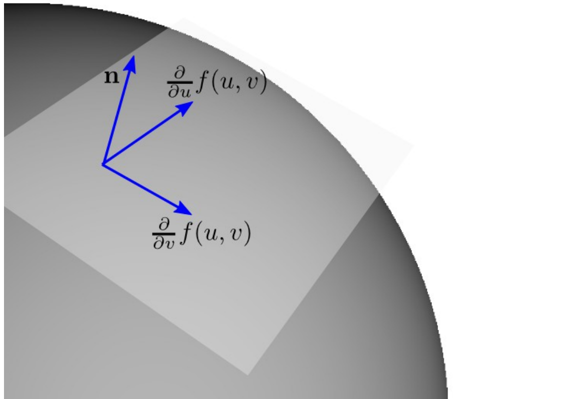
$$\text{implicit surface}: f(x, y, z) = 0$$

$$n = \frac{\nabla f}{[\nabla f]}$$

$$\text{parametric surface}:$$

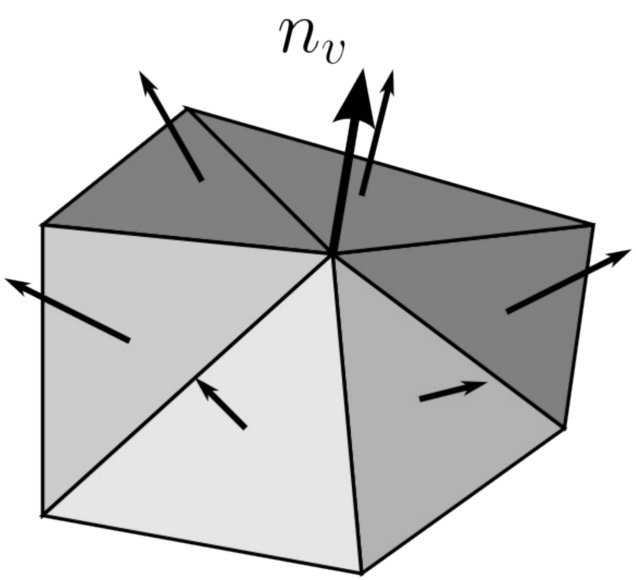
$$f(u, v) = (f_x(u, v), f_y(u, v), f_z(u, v))$$

$$n = \frac{\partial}{\partial u} f \times \frac{\partial}{\partial v} f$$



Normals on triangle meshes

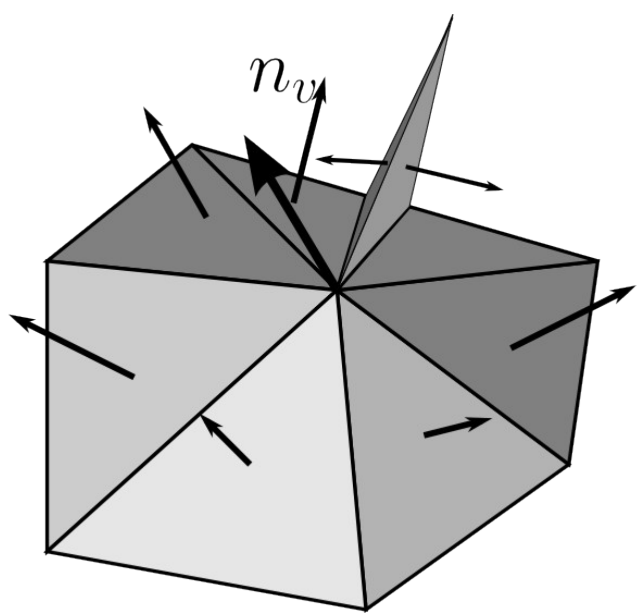
- Computed per-vertex and interpolated over the faces
- Common: consider the tangent plane as the average among the planes containing all the faces incident on the vertex



$$n_v = \frac{1}{\#N(v)} \sum_{f \in N(v)} n_f$$
$$N(v) = \{f : f \; coface \; of \; v\}$$

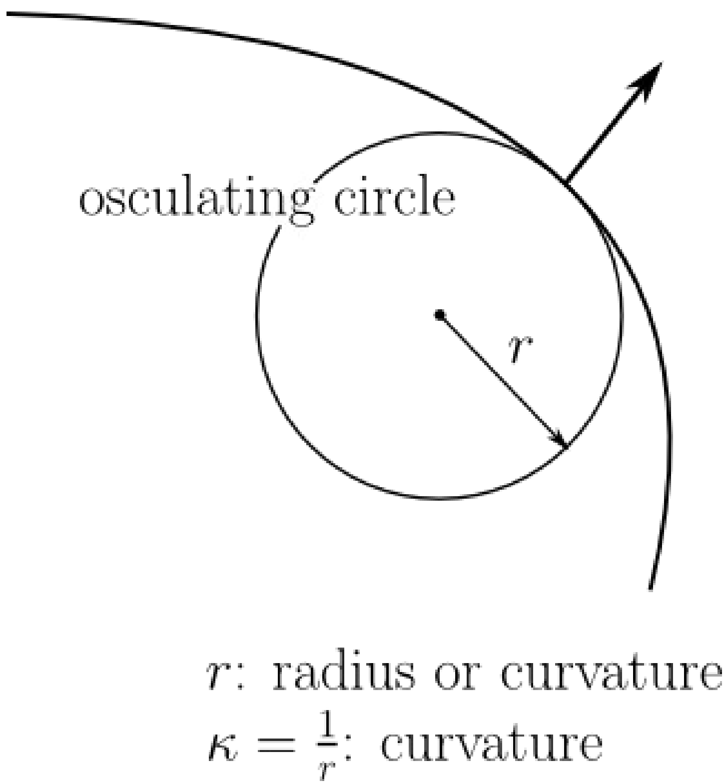
Normals on triangle meshes

- Does it work? Yes, for a “good” tessellation
  - Small triangles may change the result dramatically
  - Weighting by edge length / area / angle helps



Differential quantities: Curvature

- The curvature is a measure of how much a line is curve



$x(t), y(t)$ *a parametric curve*

φ *tangential angle*

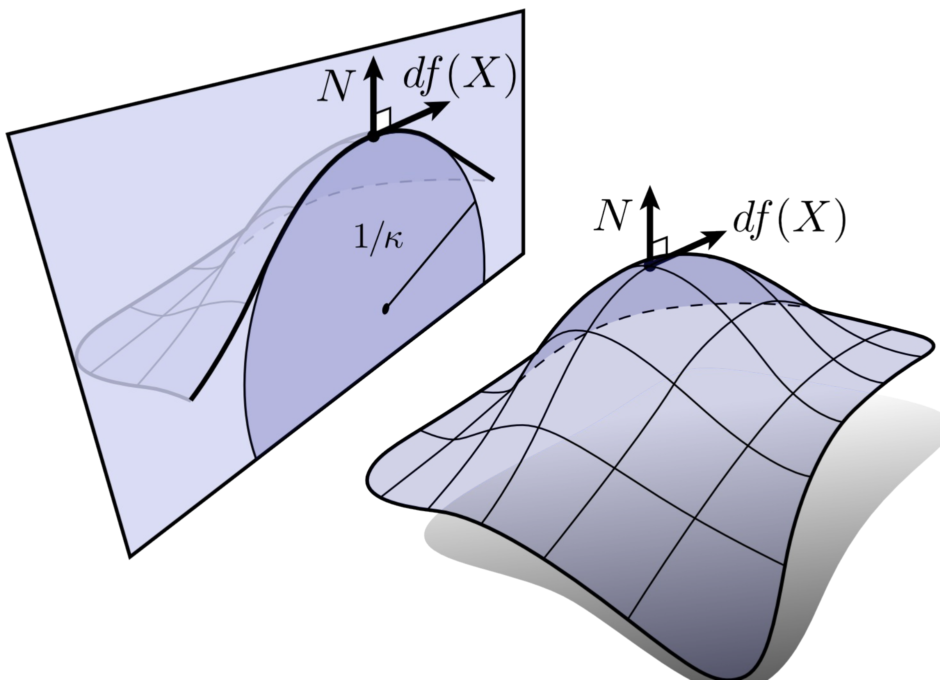
%s *arc lenght*

$$\kappa \equiv \frac{d\varphi}{ds} = \frac{d\varphi/dt}{ds/dt} = \frac{d\varphi/dt}{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}} = \frac{d\varphi/dt}{\sqrt{x'^2 + y'^2}}$$

$$\kappa = \frac{x'y'' - y'x''}{\left(x'^2 + y'^2\right)^{3/2}}$$

Curvature on a surface

- Given the normal at point *p* and a tangent direction θ
- The curvature along θ is the 2D curvature of the intersection between the plane and the surface



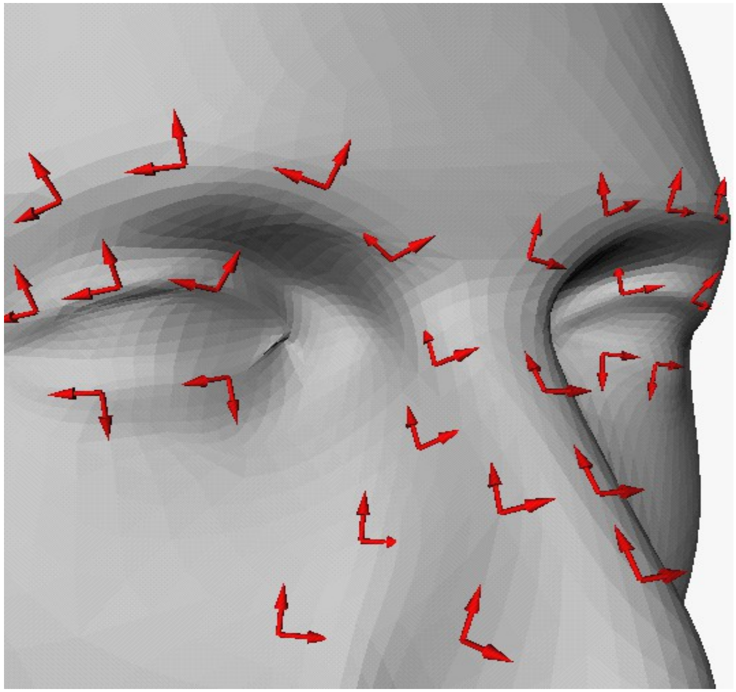
Curvatures

- A curvature for each direction
- Take the two directions for which curvature is max and min

$$\kappa_1, \kappa_2 \quad principal \quad curvatures$$

$$e_1, e_2 \quad principal \quad directions$$

- the directions of max and min curvature are orthogonal



[Meyer02]

Gaussian and Mean curvature

- Gaussian curvature: the product of principal curvatures

$$\kappa_G \equiv K \equiv \kappa_1 \cdot \kappa_2$$

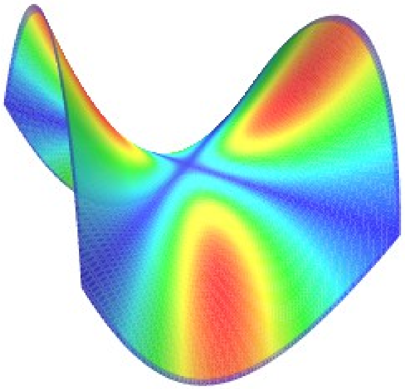
- Mean curvature: the average of principal curvatures

$$\bar{\kappa} \equiv H \equiv \frac{\kappa_1 + \kappa_2}{2}$$

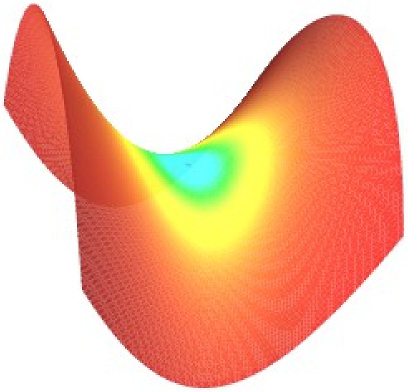
Examples..

- Red:low → red:high (not in the same scale)

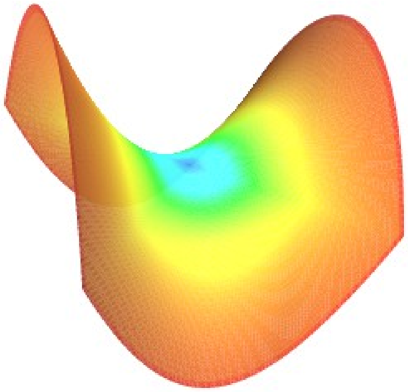
mean



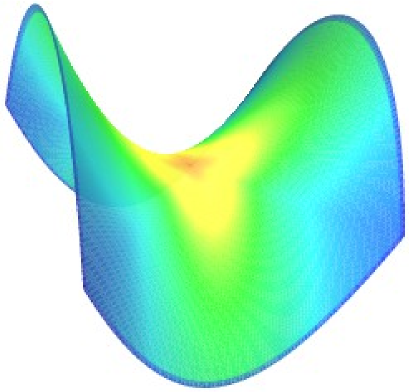
gaussian



min



max



[Meyer02]

Gaussian Curvature

- Gaussian curvature is an intrinsic property
  - It can be computed by a bidimensional inhabitant of the surface by walking around a fixed point **p** and keeping at a distance *r*.

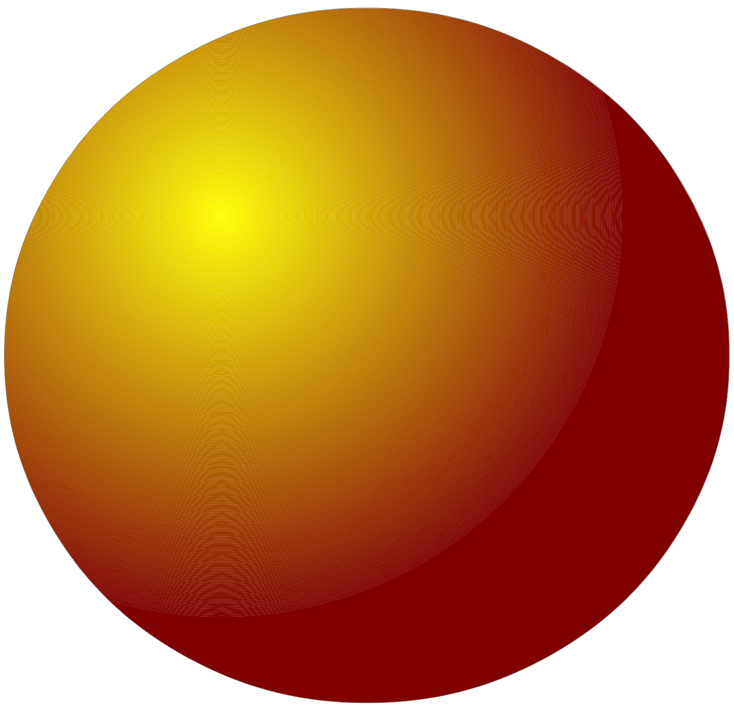
$$\kappa_G = \lim_{r \to 0} \left(2\pi r - C(r)\right) \cdot \frac{3}{\pi r^3}$$

  with *C(r)* the distance walked

- Delevopable surfaces: surfaces whose Gaussian curvature is 0 everywhere

Gaussian Curvature

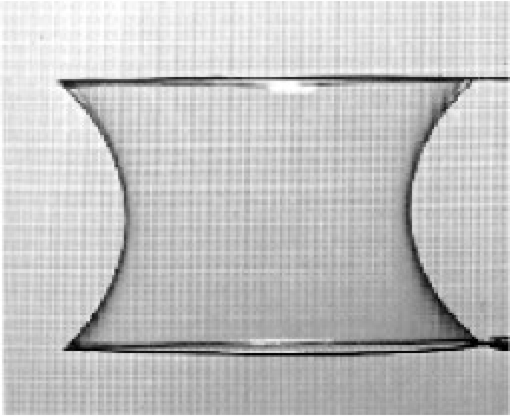
$\kappa_G > 0$

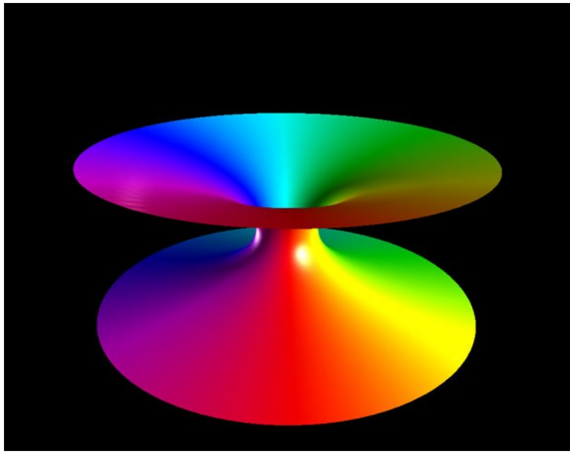


$\kappa_G < 0$

Mean Curvature

- Divergence of the surface normal
  - divergence is an operator that measures a vector field's tendency to originate from or converge upon a given point
- Minimal surface and minimal area surfaces
  - A surface is *minimal* when its mean curvature is 0 everywhere
  - All minimal area surfaces have mean curvature 0
- The surface tension of an interface, like a soap bubble, is proportional to its mean curvature





Mean Curvature

- Let A be the area of a disk around *p*. The mean curvature

$$2\bar{\kappa} n = \lim_{diam(A) \to 0} \frac{\nabla A}{A}$$

- the mean derivative is (twice) the divergence of the normal

$$2\bar{\kappa} = \nabla \cdot n = \frac{\partial}{\partial x} n + \frac{\partial}{\partial y} n + \frac{\partial}{\partial z} n$$

Gaussian curvature on a triangle mesh

- It's the *angle defect* over the area

$$\kappa_G(v_i) = \frac{1}{3A}\left(2\pi - \sum_{t_j\, adj\, v_i} \theta_j\right)$$

- **Gauss-Bonnet Theorem:** The integral of the Gaussian Curvature on a closed surface depends on the Euler number

$$\int_S \kappa_G = 2\pi\chi$$

Example data structure

- Simplest
- List of triangles:
  - For each triangle store its coords.
  -
  - 1. (3,-2,5), (3,6,2), (-6,2,4)
  - 2. (2,2,4), (0,-1,-2), (9,4,0)
  - 3. (1,2,-2), (3,6,2), (-4,-5,1)
  - 4. (-8,2,7), (-2,3,9), (1,2,-2)
- How to find any adjacency?
- Does it store FV?

Example data structure

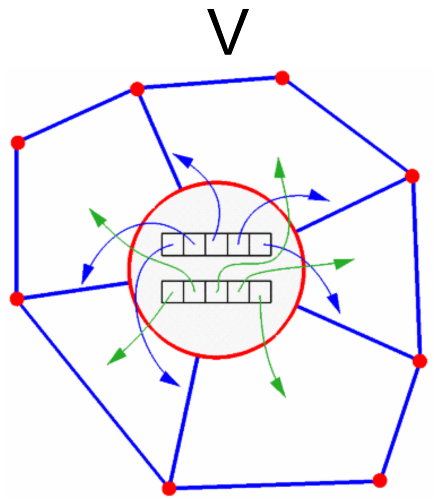
- Slightly better
- List of unique vertices with indexed faces
  - Storing the FV relation

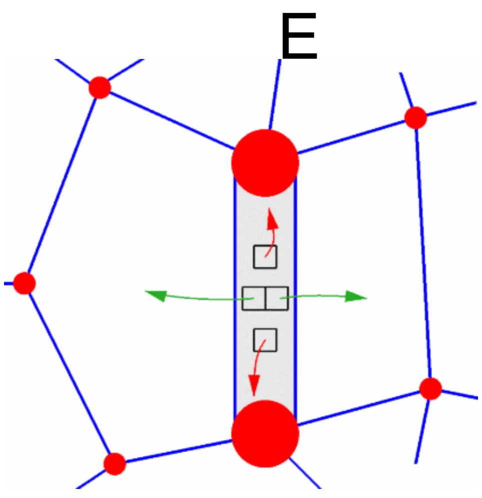
- Vertices:
  1. (-1.0, -1.0, -1.0)
  2. (-1.0, -1.0, 1.0)
  3. (-1.0, 1.0, -1.0)
  4. (-1, 1, 1.0)
  5. (1.0, -1.0, -1.0)
  6. (1.0, -1.0, 1.0)
  7. (1.0, 1.0, -1.0)
  8. (1.0, 1.0, 1.0)

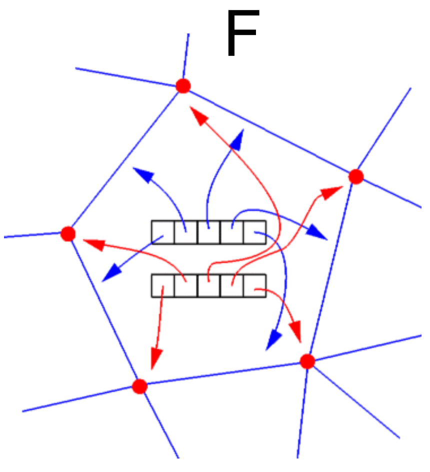
- Faces:
  1. 1 2 4
  2. 5 7 6
  3. 1 5 2
  4. 3 4 7
  5. 1 7 5

Example data structure

- Issue of Adjacency
  - Vertex, Edge, and Face Structures
    - Each element has list of pointers to all incident elements
    - Queries depend only on local complexity of mesh!
    - Slow! Big! Too much work to maintain!
    - Data structures do not have fixed size

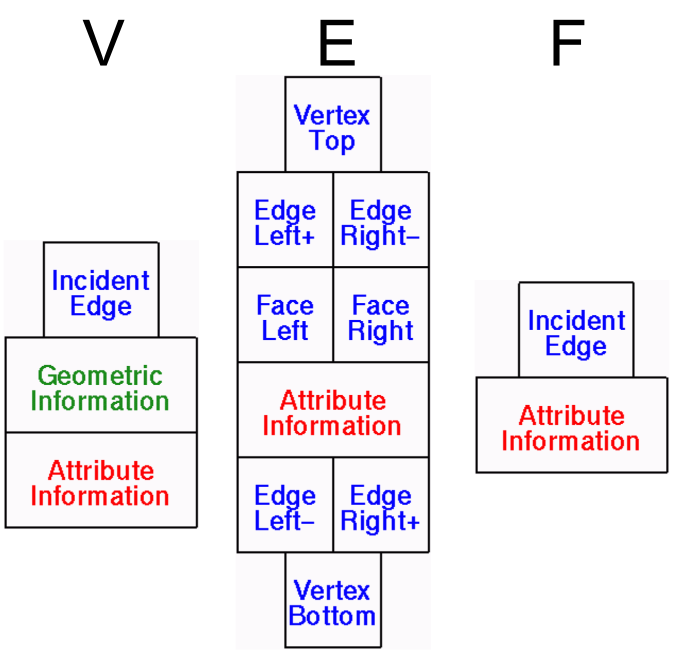


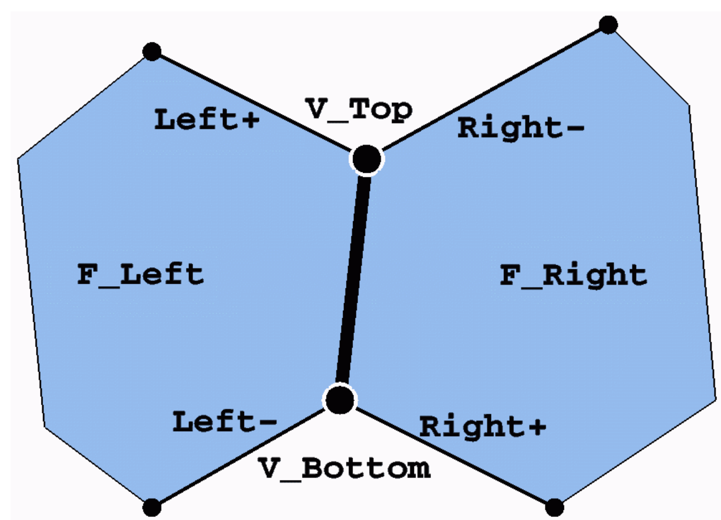




Example data structure

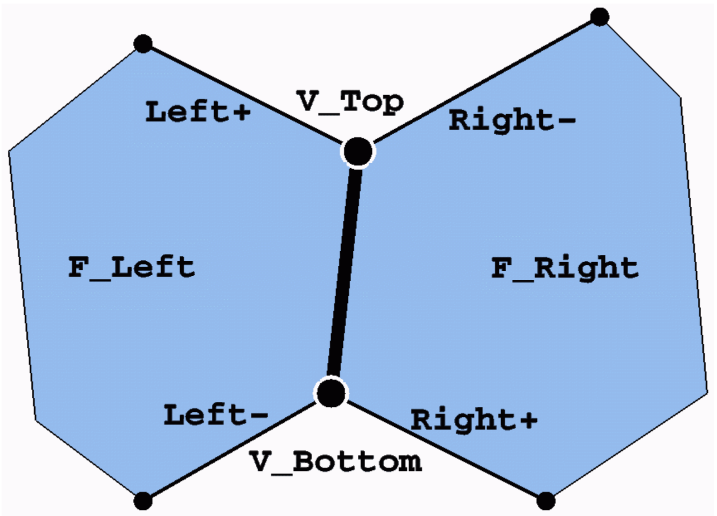
- Winged edge
  - Classical real smart structure
  - Nice for generic polygonal meshes
  - Used in many sw packages





Winged Edge

- Winged edge
  - Compact
  - All the query requires some kind of "traversal"
  - Not fitted for rendering...



Data structures for 3D Meshes

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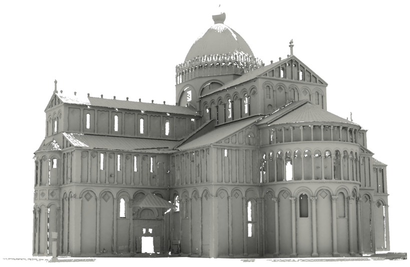
http://vcg.isti.cnr.it/~cignoni

epresenting Real World Surfaces

nalytic definition falls short of epresenting *real world* surfaces in a *ractable* way

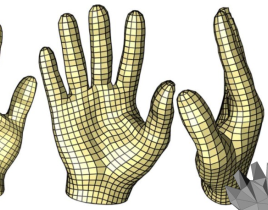
$S(x, y) = \ldots?$



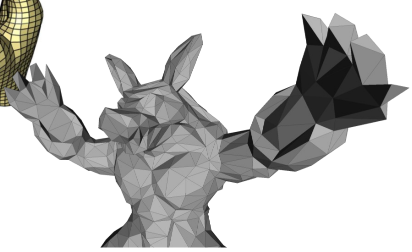
.. surfaces can be represented by ***cell complexes***

Cell complexes (meshes)

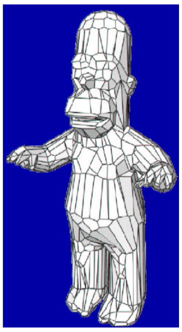
ntuitive description: a continuous
rface divided in polygons



adrilaterals (quads)



triangles

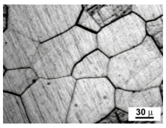


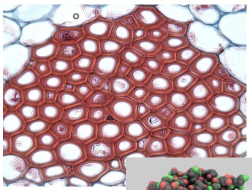
Generic polygons

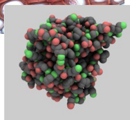
Cell Complexes (meshes)

n nature, meshes arise in a variety of ontexts:

- Cells in organic tissues
- Crystals
- Molecules
- …





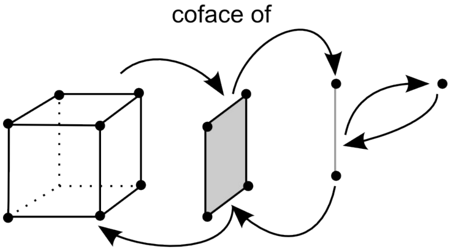


- Mostly *convex* but *irregular* cells
- Common concept: *complex* shapes can be described as *collections* of *simple building blocks*

Cell Complexes (meshes)

more formal definition

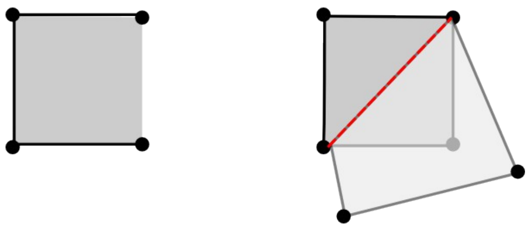
- a *cell* is a convex polytope in
- a *proper face* of a cell is a convex polytope in



Cell Complexes (meshes)

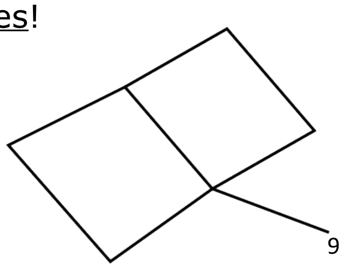
a collection of cells is a complex **iff**

- every face of a cell belongs to the complex
- For every cells C and C′, their intersection either is empty or is a common face of both



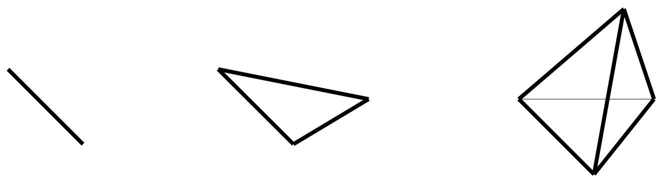
Maximal Cell Complex

- the **order** of a cell is the number of its sides (or ertices)
- a complex is a **k-complex** if the maximum of the rder of its cells is *k*
- a cell is **maximal** if it is not a face of another cell
- a k-complex is **maximal** *iff* all maximal cells have rder k
- short form : no dangling edges!



Simplicial Complex

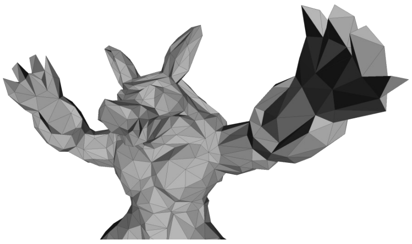
- A cell complex is a **simplicial complex** when the cells are simplexes
- A **d-*simplex*** is the convex hull of *d+1* points in



mplex 1-simplex 2-simplex 3-simplex

Meshes, at last

When talking of *triangle mesh* the intended meaning is a **maximal 2-simplicial complex**



Topology vs Geometry

- Di un complesso simpliciale e' buona norma distinguere
  - Realizzazione geometrica
    - Dove stanno effettivamente nello spazio i vertici del nostro complesso simpliciale
  - Caratterizzazione topologica
    - Come sono connessi combinatoriamente i vari elementi

Topology vs geometry 2

Nota: Di uno stesso oggetto e' possibile dare rappresentazioni con eguale realizzazione geometrica ma differente caratterizzazione topologica (molto differente!) Demo kleine

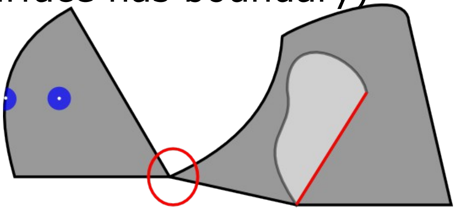
Nota: Di un oggetto si puo' dire molte cose considerandone solo la componente topologica

- Orientabilita
- componenti connesse
- bordi

Manifoldness

a surface S is **2-manifold** *iff:*

- the neighborhood of each point is homeomorphic to Euclidean space in two dimension
  or … in other words..
- the neighborhood of each point is homeomorphic to a disk (or a semidisk if the surface has boundary)

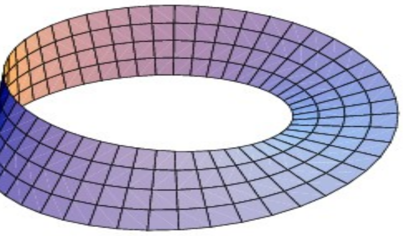


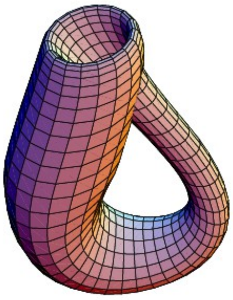
Orientability

A surface is **orientable** if it is possible to
nake a consistent choice for the normal vector

- …it has two sides

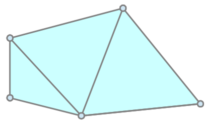
oebius strips, klein bottles, and non manifold
urfaces are not orientable





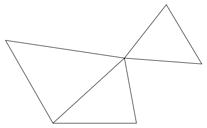
Incidenza Adiacenza

- Due simplessi σ e σ' sono incidenti se σè una faccia propria di σ' o vale il viceversa.
- Due k-simplessi sono m-adiacenti (k>m) se esiste un m-simplesso che è una faccia propria di entrambi.
  - Due triangoli che condividono un edge sono 1 -adiacenti
  - Due triangoli che condividono un vertice sono 0-adiacenti



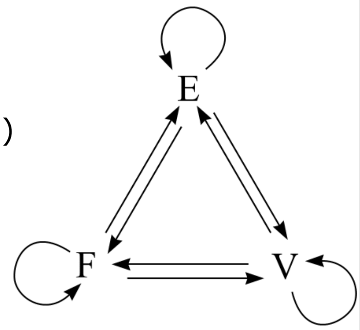
Relazioni di Adiacenza

- Per semplicità nel caso di mesh si una relazione di adiacenza con un una coppia (ordinata!) di lettere che indicano le entità coinvolte
  - FF adiacenza tra triangoli
  - FV i vertici che compongono un triangolo
  - VF i triangoli incidenti su un dato vertice



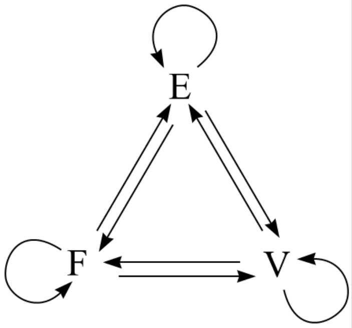
Relazioni di adiacenza

- Di tutte le possibili relazioni di adiacenza di solito vale la pena se ne considera solo un sottoinsieme (su 9) e ricavare le altre proceduralmente



Relazioni di adiacenza

- FF ~ 1-adiacenza
- EE ~ 0 adiacenza
- FE ~ sottofacce proprie di F con dim 1
- FV ~ sottofacce proprie di F con dim 0
- EV ~ sottofacce proprie di E con dim 0
- VF ~ F in Σ : V sub faccia di F
- VE ~ E in Σ : V sub faccia di E
- EF ~ F in Σ : E sub faccia di F
- VV ~ V' in Σ : Esiste E (V,V')



Partial adiacency

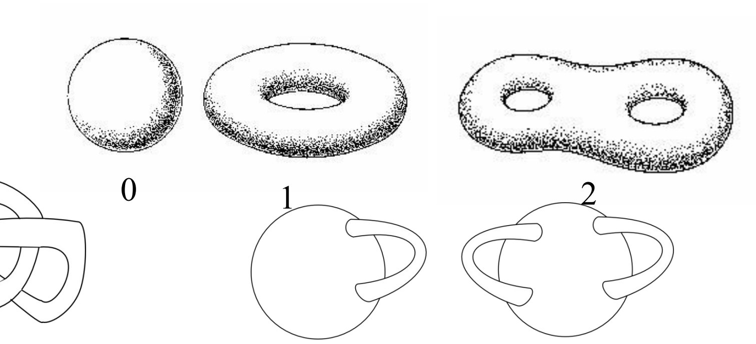
- Per risparmiare a volte si mantiene una informazione di adiacenza parziale
  - VF* memorizzo solo un riferimento dal vertice ad una delle facce e poi 'navigo' sulla mesh usando la FF per trovare le altre facce incidenti su V

Relazioni di adiacenza

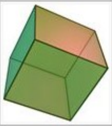
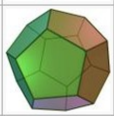
- In un 2-complesso simpliciale immerso in R3, che sia 2 manifold
  - FV FE FF EF EV sono di cardinalità bounded (costante nel caso non abbia bordi)
    - |FV|= 3 |EV| = 2 |FE| = 3
    - |FF| <= 2
    - |EF| <= 2
  - VV VE VF EE sono di card. variabile ma in stimabile in media
    - |VV|~|VE|~|VF|~6
    - |EE|~10
    - F ~ 2V

Genus

ne **Genus** of a closed surface, orientable and 2-manifold
ne maximum number of cuts we can make along non
ersecting closed curves without splitting the surface in
.

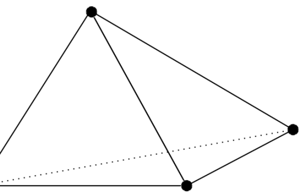


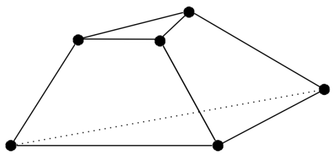
also known as the number of *handles*

Name	Image	V (vertices)	E (edges)	F (faces)	Euler characteristic: $V - E + F$
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
cosahedron		12	30	20	**2**

Euler characteristics

- (= 2 for any *simply connected* polyhedron
- roof by construction…
- lay with examples:



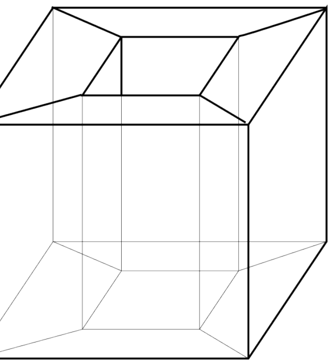


$V - E + F$
$4 - 6 + 4 = 2$

$\chi = (V+2) - (E+3) + (F+1) =$
$\chi = (4+2) - (6+3) + (4+1) = 2$

Euler characteristics

let’s try a more complex figure…



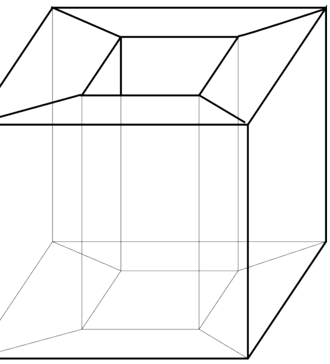
$\chi = V - E + F$
$\chi = 16 - 32 + 16 = \mathbf{0}$

why =0 ?

Euler characteristics

$$\chi = 2 - 2g$$

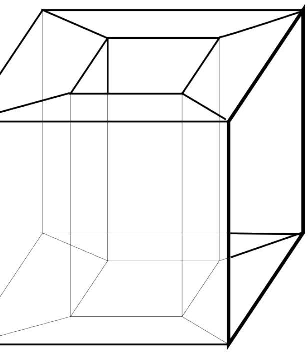
where *g* is the genus of the surface



$\chi = V - E + F$
$\chi = 16 - 32 + 16 = 0 = 2 - 2g$

Euler characteristics

let's try a more complex figure…remove a
ace. The surface is not closed anymore



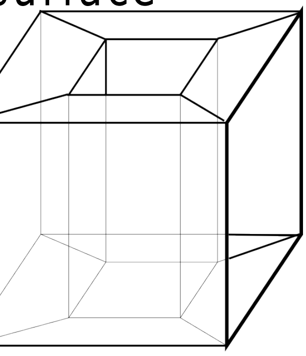
$$\chi = V \quad - E + F$$
$$\chi = 16 \ - 32 \ + 15 = -\mathbf{1}$$

why =-1 ?

Euler characteristics

$$\chi = 2 - 2g - b$$

where b is the number of borders of the surface

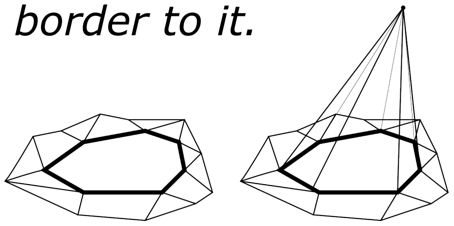


$\chi = V - E + F$

$\chi = 16 - 32 + 15 = -1 = 2 - 2g - b$

Euler characteristics

*Remove the border by adding a new vertex and connecting all the **k** vertices on the border to it.*



A A’

$X' = X + V' - E' + F' = X + 1 - k + k = X + 1$

Differential quantities:normals

- The (unit) **normal** to a point is the (unit) vector perpendicular to the tangent plane

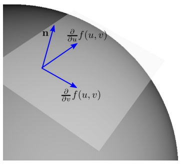
$$\text{implicit surface}: f(x, y, z) = 0$$

$$n = \frac{\nabla f}{[\nabla f]}$$

$$\text{parametric surface}:$$

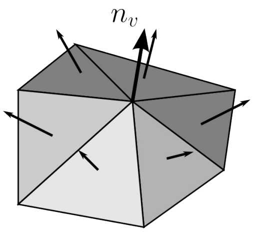
$$f(u, v) = (f_x(u, v), f_y(u, v), f_z(u, v))$$

$$n = \frac{\partial}{\partial u} f \times \frac{\partial}{\partial v} f$$



Normals on triangle meshes

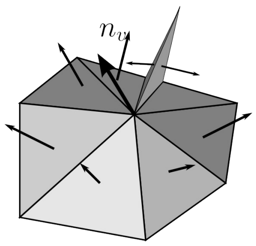
- Computed per-vertex and interpolated over the faces
- Common: consider the tangent plane as the average among the planes containing all the faces incident on the vertex



$$n_v = \frac{1}{\#N(v)} \sum_{f \in N(v)} n_f$$
$$N(v) = \{f : f \ coface \ of \ v\}$$

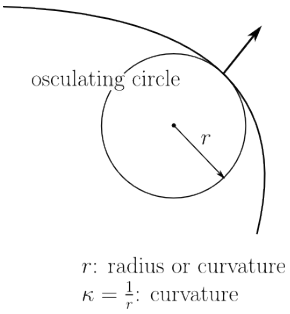
Normals on triangle meshes

- Does it work? Yes, for a “good” tessellation
  - Small triangles may change the result dramatically
  - Weighting by edge length / area / angle helps



Differential quantities: Curvature

- The curvature is a measure of how much a line is curve



$x(t), y(t)$ *a parametric curve*

φ *tangential angle*

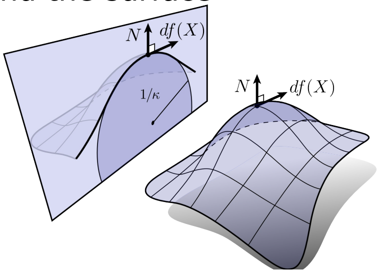
%s arc lenght

$$\kappa \equiv \frac{d\varphi}{ds} = \frac{d\varphi/dt}{ds/dt} = \frac{d\varphi/dt}{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}} = \frac{d\varphi/dt}{\sqrt{x'^2 + y'^2}}$$

$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$

Curvature on a surface

- Given the normal at point *p* and a tangent direction θ
- The curvature along θ is the 2D curvature of the intersection between the plane and the surface

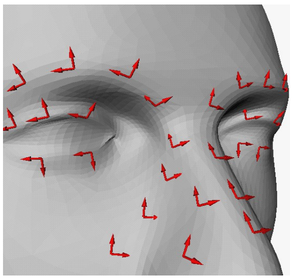


Curvatures

- A curvature for each direction
- Take the two directions for which curvature is max and min

$$\kappa_1, \kappa_2 \quad principal \quad curvatures$$
$$e_1, e_2 \quad principal \quad directions$$

- the directions of max and min curvature are orthogonal



[Meyer02]

Gaussian and Mean curvature

- Gaussian curvature: the product of principal curvatures

$$\kappa_G \equiv K \equiv \kappa_1 \cdot \kappa_2$$

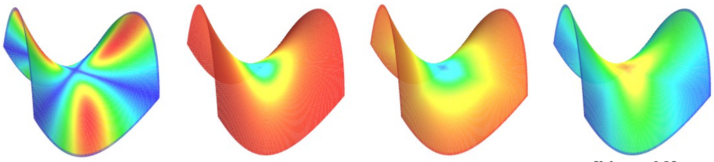
- Mean curvature: the average of principal curvatures

$$\bar{\kappa} \equiv H \equiv \frac{\kappa_1 + \kappa_2}{2}$$

Examples..

- Red:low → red:high (not in the same scale)

mean gaussian min max



[Meyer02]

Gaussian Curvature

- Gaussian curvature is an intrinsic property
  - It can be computed by a bidimensional inhabitant of the surface by walking around a fixed point **p** and keeping at a distance *r.*

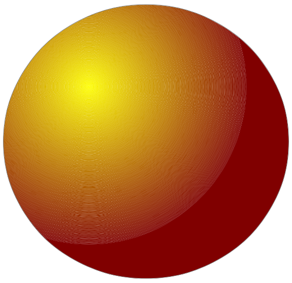
$$\kappa_G = \lim_{r \to 0} \left(2\pi r - C(r)\right) \cdot \frac{3}{\pi r^3}$$

  with *C(r)* the distance walked

- Delevopable surfaces: surfaces whose Gaussian curvature is 0 everywhere

Gaussian Curvature

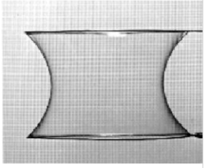
$\kappa_G > 0$

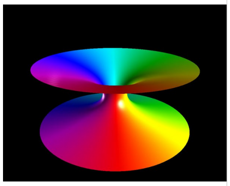


$\kappa_G < 0$

Mean Curvature

- Divergence of the surface normal
  - divergence is an operator that measures a vector field's tendency to originate from or converge upon a given point
- Minimal surface and minimal area surfaces
  - A surface is *minimal* when its mean curvature is 0 everywhere
  - All minimal area surfaces have mean curvature 0
- The surface tension of an interface, like a soap bubble, is proportional to its mean curvature





Mean Curvature

- Let A be the area of a disk around *p.* The mean curvature

$$2\bar{\kappa}n = \lim_{diam(A)\to 0} \frac{\nabla A}{A}$$

- the mean derivative is (twice) the divergence of the normal

$$2\bar{\kappa} = \nabla \cdot n = \frac{\partial}{\partial x}n + \frac{\partial}{\partial y}n + \frac{\partial}{\partial z}n$$

Gaussian curvature on a triangle mesh

- It's the *angle defect* over the area

$$\kappa_G(v_i)=\frac{1}{3A}(2\pi-\sum_{t_j\,adj\,v_i}\theta_j)$$

- **Gauss-Bonnet Theorem:** The integral of the Gaussian Curvature on a closed surface depends on the Euler number

$$\int_S \kappa_G = 2\pi\chi$$

Example data structure

- Simplest
- List of triangles:
  - For each triangle store its coords.
  -
  - 1. (3,-2,5), (3,6,2), (-6,2,4)
  - 2. (2,2,4), (0,-1,-2), (9,4,0)
  - 3. (1,2,-2), (3,6,2), (-4,-5,1)
  - 4. (-8,2,7), (-2,3,9), (1,2,-2)
- How to find any adjacency?
- Does it store FV?

Example data structure

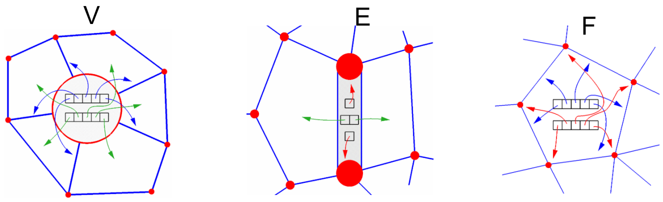
- Slightly better
- List of unique vertices with indexed faces
  - Storing the FV relation

- Vertices:
  - 1. (-1.0, -1.0, -1.0)
  - 2. (-1.0, -1.0, 1.0)
  - 3. (-1.0, 1.0, -1.0)
  - 4. (-1, 1, 1.0)
  - 5. (1.0, -1.0, -1.0)
  - 6. (1.0, -1.0, 1.0)
  - 7. (1.0, 1.0, -1.0)
  - 8. (1.0, 1.0, 1.0)

- Faces:
  - 1. 1 2 4
  - 2. 5 7 6
  - 3. 1 5 2
  - 4. 3 4 7
  - 5. 1 7 5

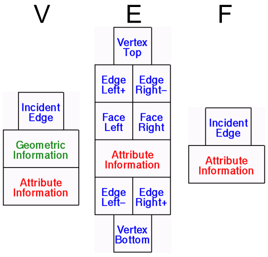
Example data structure

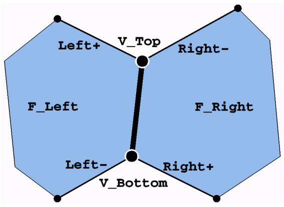
- Issue of Adjacency
  - Vertex, Edge, and Face Structures
    - Each element has list of pointers to all incident elements
    - Queries depend only on local complexity of mesh!
    - Slow! Big! Too much work to maintain!
    - Data structures do not have fixed size



Example data structure

- Winged edge
  - Classical real smart structure
  - Nice for generic polygonal meshes
  - Used in many sw packages





Winged Edge

- Winged edge
  - Compact
  - All the query requires some kind of "traversal"
  - Not fitted for rendering...